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The $\mu \to e\gamma$ and $\tau \to \mu\gamma$ decays in the general two Higgs doublet model with the inclusion of one universal extra dimension

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Abstract: We study the effect of one universal extra dimension on the branching ratios of the lepton flavor violating processes $\mu \to e\gamma$ and $\tau \to \mu\gamma$ in the general two Higgs doublet model. We observe that these new effects are tiny for the small values of the compactification radius R. Furthermore, we see that these effects are comparable with the branching ratio obtained without including extra dimension, if the neutral Higgs bosons are nearly degenerate and the complexity of the Yukawa coupling, inducing the vertex tau-tau-h^0 (A^0), is large.

Keywords: Beyond Standard Model, Rare Decays.

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Contents

1. Introduction

Lepton Flavor Violating (LFV) interactions are rich from the theoretical point of view since they exist at the loop level and the related measurable quantities contain number of free parameters of the model used. In such decays, the assumption of the non-existence of Cabibbo-Kobayashi-Maskawa (CKM) type matrix in the leptonic sector, in the framework of the standard model (SM), forces one to search the physics beyond. The improvement of the experimental measurements of the LFV processes make it possible to understand the new physics effects more accurately. One of the candidate model beyond the SM is the general two Higgs doublet model (2HDM), so called the model III. In this model the LFV interactions are induced by the internal neutral Higgs bosons h^0 and A^0 and the Yukawa couplings, appearing as free parameters, can be determined by the experimental data. The $\mu \to e\gamma$ and $\tau \to \mu\gamma$ are the examples of LFV interactions and the current limits for their branching ratios (BR's) are 1.2×10^{-11} [1] and 1.1×10^{-6} [2] respectively. In the literature, there are several studies on the LFV interactions in various models. Such interactions are investigated in a model independent way in [3], in supersymmetric models [4], in the framework of the 2HDM [5, 6].

In this work, we study the LFV processes $\mu \to e\gamma$ and $\tau \to \mu\gamma$ in the framework of the model III, with the addition of one extra spatial dimension. Higher dimensional scenarios has been induced by the string theories as a possible solution to the hierarchy problem of the standard model (SM) and there is an extensive work on this subject in the literature [7]–[16]. The idea is that the ordinary four dimensional SM is the low energy effective theory of a more fundamental one lying in larger dimensions and the extra dimensions over the 4-dimension are compactified on a circle of a radius R, which is a typical size of an extra dimension. This size has been studied using the present experimental data in several works [9, 10] and estimated as large as few hundereds of GeV [7, 8, 11], not to contradict with the experiments. Furthermore, the loop effects induced by the internal top quark are sensitive to the KK mode contributions and the size of the extra dimensions has been estimated in the range $200 - 500\,\mathrm{GeV}$, using electroweak precision measurements [12], the $B - \bar{B}$ -mixing [13, 14] and the flavor changing process $b \to s\gamma$ [15].

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In the 4 dimensions, the extra dimension takes the form of Kaluza-Klein (KK) excitations of fields, 2HDM fields in our case, with masses $\sim n/R$. In the case that all 2HDM fields lie in the extra dimension, the extra dimensional momentum is conserved and, in our case, the coupling of Higgs boson-lepton-lepton vertex involves one external zero mode lepton and internal KK mode of lepton and neutral Higgs boson. Such extra dimensions are called universal extra dimensions (UED).

In the LFV $\mu \to e\gamma$ and $\tau \to \mu\gamma$ decays, the effect of one UED is carried by the KK mode of internal neutral Higgs fields, h^0, A^0, and internal lepton fields, at one loop order, in the model III version of the 2HDM. The non-zero KK modes of neutral Higgs fields H have masses $\sqrt{m_H^2 + m_n^2}$ with $m_n = n/R$. Here $m_n = n/R$ is the mass of n'th level KK particle where R is the compactification radius. Similarly, the non-zero KK modes of lepton doublets (singlets) have the left (right) handed lepton fields with masses $\sqrt{m_{l_i}^2 + m_n^2}$ and the right (left) handed ones with masses m_n. It should be noted that the KK spectrum at each excitation level is nearly degenerate for different lepton flavors since their masses are smaller compared to the mass $m_n = n/R$.

This work is devoted to the the effect of one UED on the BR of the LFV processes $\mu \to e\gamma$ and $\tau \to \mu\gamma$ in the framework of the model III. It is observed that these new effects are tiny for the small values of the compactification radius. Furthermore, these effects are comparable with the BR obtained without including extra dimension, in the case that the neutral Higgs bosons are nearly degenerate and the complexity of the Yukawa coupling is large.

The paper is organized as follows: In section 2, we present the BR's of LFV interactions $\mu \to e\gamma$ and $\tau \to \mu\gamma$ in the model III version of the 2HDM with the inclusion of one universal extra dimension. Section 3 is devoted to discussion and our conclusions.

2. The LFV interactions $\mu \to e\gamma$ and $\tau \to \mu\gamma$ in the general two Higgs doublet model with the inclusion of one universal extra dimension

In the model III version of the 2HDM the flavor changing neutral currents (FCNC) at tree level is permitted and the LFV interactions exist with larger BR‘s, compared to ones obtained in the SM. The main parameters in this calculation are the new Yukawa couplings, that can be chosen complex. The addition of one spatial UED brings new contributions to the BR‘s of LFV processes at one loop order and the part of the lagrangian responsible for these interactions is the Yukawa lagrangian, which reads in 5 dimension

$$\mathcal{L}_Y = \eta^E_{5\,ij} \bar{l}_i \phi_1 E_j + \xi^E_{5\,ij} \bar{l}_i \phi_2 E_j + \text{h.c.}\,, \tag{2.1}$$

with 5-dimensional Yukawa couplings $\eta^E_{5\,ij}$, $\xi^E_{5\,ij}$, where i, j are family indices of leptons, ϕ_i for $i = 1, 2$, are the two scalar doublets, l_i and E_j are lepton doublets and singlets respectively. These fields are the functions of x^μ and y, where y is the coordinate represents the 5‘th dimension. The Yukawa couplings $\eta^E_{5\,ij}$, $\xi^E_{5\,ij}$ are dimensionful and rescaled to the ones in 4-dimension as $\eta^E_{5\,ij} = \sqrt{2\pi R}\,\eta^E_{ij}$, and $\xi^E_{5\,ij} = \sqrt{2\pi R}\,\xi^E_{ij}$. Here ϕ_1 and ϕ_2 are

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chosen as

$$\phi_1 = \frac{1}{\sqrt{2}}\left[\begin{pmatrix} 0 \\ v + H^0 \end{pmatrix} + \begin{pmatrix} \sqrt{2}\chi^+ \\ i\chi^0 \end{pmatrix}\right]; \qquad \phi_2 = \frac{1}{\sqrt{2}}\begin{pmatrix} \sqrt{2}H^+ \\ H_1 + iH_2 \end{pmatrix}, \tag{2.2}$$

and the vacuum expectation values are

$$\langle\phi_1\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v \end{pmatrix}; \qquad \langle\phi_2\rangle = 0\,. \tag{2.3}$$

With this choice, the SM particles can be collected in the first doublet and the new particles in the second one. The part which produce FCNC at tree level is

$$\mathcal{L}_{Y,FC} = \xi^E_{5\,ij}\bar{l}_i\phi_2 E_j + \text{h.c.}\,. \tag{2.4}$$

Here the Yukawa matrices $\xi^E_{5\,ij}$ have in general complex entries and they are the source of CP violation. Note that in the following we replace ξ^E with ξ^E_N where "N" denotes the word "neutral". The five dimensional lepton doublets and singlets have both chiralities and the 4-dimensional lagrangian is constructed by expanding these fields into their KK modes. Besides, the extra dimension denoted by y is compactified on a circle of radius R and the zero modes of the wrong chirality (l_{iR}, E_{iL}) are projected out by compactification of the fifth dimension y on an S^1/Z_2 orbifold, namely $Z_2 : y \to -y$. The KK decompositions of the lepton and Higgs fields read

$$\begin{aligned}
\phi_{1,2}(x,y) &= \frac{1}{\sqrt{2\pi R}}\left\{\phi^{(0)}_{1,2}(x) + \sqrt{2}\sum_{n=1}^{\infty}\phi^{(n)}_{1,2}(x)\cos\left(\frac{ny}{R}\right)\right\} \\
l_i(x,y) &= \frac{1}{\sqrt{2\pi R}}\left\{l^{(0)}_{iL}(x) + \sqrt{2}\sum_{n=1}^{\infty}\left[l^{(n)}_{iL}(x)\cos\left(\frac{ny}{R}\right) + l^{(n)}_{iR}(x)\sin\left(\frac{ny}{R}\right)\right]\right\} \\
E_i(x,y) &= \frac{1}{\sqrt{2\pi R}}\left\{E^{(0)}_{iR}(x) + \sqrt{2}\sum_{n=1}^{\infty}\left[E^{(n)}_{iR}(x)\cos\left(\frac{ny}{R}\right) + E^{(n)}_{iL}(x)\sin\left(\frac{ny}{R}\right)\right]\right\},
\end{aligned} \tag{2.5}$$

where $\phi^{(0)}_{1,2}(x)$, $l^{(0)}_{iL}(x)$ and $E^{(0)}_{iR}(x)$ are the 4-dimensional Higgs doublets, lepton doublets and lepton singlets respectively. Here L and R denote chiral projections $L(R) = 1/2(1\mp\gamma_5)$ and they are four dimensional. Each non-zero KK mode of Higgs doublet ϕ_2 (ϕ_1) includes a charged Higgs of mass $\sqrt{m^2_{H^\pm}+m^2_n}$ (m_n), a neutral CP even Higgs of mass $\sqrt{m^2_{h^0}+m^2_n}$ ($\sqrt{m^2_{H^0}+m^2_n}$), a neutral CP odd Higgs of mass $\sqrt{m^2_{A^0}+m^2_n}$ (m_n) with $m_n = n/R$. Here $m_n = n/R$ is the mass of n'th level KK particle where R is the compactification radius. Similarly, the non-zero KK modes of lepton doublets (singlets) have the left (right) handed lepton fields with masses $\sqrt{m^2_{l_i}+m^2_n}$ and the right (left) handed ones with masses m_n. Notice that the KK spectrum at each excitation level is nearly degenerate for different lepton flavors since their masses are smaller compared to the mass $m_n = n/R$.

Now we will consider the lepton flavor violating processes $\mu\to e\gamma$ and $\tau\to\mu\gamma$ with the addition of one spatial extra dimension. These processes are good candidates for searching the new physics beyond the SM and for the determination of the free parameters existing.

Here we take into account only the neutral Higgs contributions in the lepton sector of the model III and, therefore, the neutral Higgs bosons h^0 and A^0 are responsible for these interactions. The addition of one extra spatial dimension brings new contribution due to the internal h^{0n}- l_i^n ($l_{1,2,3} = \tau, \mu, e$), A^{0n}- l_i^n KK modes and these contributions are calculated by taking the vertices involving one zero mode and two non-zero modes (see figure 1). The processes we consider exist at least in the one loop level and the corresponding diagrams are classified in two different groups, namely, self energy diagrams and vertex diagrams. Now, we use the on-shell renormalization scheme and extract the nonvanishing structures to get a gauge invariant result. In this scheme, the self energy diagrams for on-shell leptons vanish since they can be written as

$$\sum(p) = (\hat{p} - m_{l_1})\bar{\sum}(p)(\hat{p} - m_{l_2})\,. \tag{2.6}$$

However, the vertex diagrams a and b in figure 1 give non-zero contribution. The bare vertex function Γ^0_α has a logarithmic divergence and it can be eliminated by introducing the counter term Γ^C_α to satisfy the expression

$$k^\alpha \Gamma^{\rm Ren}_\alpha = 0\,, \tag{2.7}$$

where $\Gamma^{\rm Ren}_\alpha$ is $\Gamma^{\rm Ren}_\alpha = \Gamma^0_\alpha + \Gamma^C_\alpha$ and it is in the form

$$\Gamma^{\rm Ren}_\alpha = 2\,A_1\,(B_1)\,\sigma_{\alpha\nu}\,k^\nu R + 2\,A_2\,(B_2)\,\sigma_{\alpha\nu}\,k^\nu L\,. \tag{2.8}$$

Here k_ν is the photon momentum 4-vector, $A_1(B_1)$ and $A_2\,(B_2)$ are the functions of the masses of the leptons and the Yukawa couplings (see eq. (2.10) and (2.12)) appearing in $\mu \to e\gamma$ ($\tau \to \mu\gamma$) decay.

Using the matrix element $M = \epsilon^\alpha \bar{e}\Gamma^{\rm Ren}_\alpha \mu$, with the polarization vector ϵ^α of the photon, the decay width for the process $\mu \to e\gamma$ can be obtained as

$$\Gamma(\mu \to e\gamma) = c_1 \left(|A_1|^2 + |A_2|^2\right), \tag{2.9}$$

where

$$\begin{aligned} A_1 &= Q_\tau \frac{1}{48 m_\tau^2}\Bigg(6 m_\tau \bar{\xi}^{E*}_{N,\tau e}\bar{\xi}^{E*}_{N,\tau\mu}\Big(F(z_{h^0}) - F(z_{A^0})\Big) + m_\mu \bar{\xi}^{E*}_{N,\tau e}\bar{\xi}^{E}_{N,\tau\mu}\,\times \\ &\qquad \times\Bigg(G(z_{h^0}) + G(z_{A^0}) + \sum_{n=1}^{\infty}\frac{m_\tau^2}{(m_\tau^{\rm extr})^2}(G(z_{n,h^0}) + G(z_{n,A^0}))\Bigg)\Bigg)\,, \\ A_2 &= Q_\tau \frac{1}{48 m_\tau^2}\Bigg(6 m_\tau \bar{\xi}^{D}_{N,\tau e}\bar{\xi}^{D}_{N,\tau\mu}\Big(F(z_{h^0}) - F(z_{A^0})\Big) + m_\mu \bar{\xi}^{D}_{N,\tau e}\bar{\xi}^{D*}_{N,\tau\mu}\,\times \\ &\qquad \times\Bigg(G(z_{h^0}) + G(z_{A^0}) + \sum_{n=1}^{\infty}\frac{m_\tau^2}{(m_\tau^{\rm extr})^2}(G(z_{n,h^0}) + G(z_{n,A^0}))\Bigg)\Bigg)\,, \end{aligned} \tag{2.10}$$

$c_1 = \frac{G_F^2 \alpha_{em} m_\mu^3}{32\pi^4}$, and the amplitudes A_1 and A_2 have right and left chirality respectively (see eq. (2.8)). Here we take only τ lepton for the internal line.

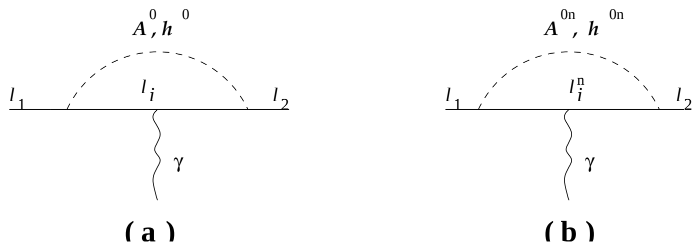

Figure 1: One loop diagrams contribute to $l_1 \to l_2 \gamma$ decay due to the zero mode (KK mode) neutral Higgs bosons h^0 and A^0 (h^{0n} and A^{0n}) in the 2HDM.

The decay width of the other LFV process $\tau \to \mu\gamma$ can be calculated using the same procedure, namely, using the matrix element $M = \epsilon^\alpha \, \bar{\mu} \, \Gamma^{\mathrm{Ren}}_\alpha \tau$, and reads as

$$\Gamma(\tau \to \mu\gamma) = c_2(|B_1|^2 + |B_2|^2) \,, \tag{2.11}$$

where

$$\begin{aligned} B_1 &= Q_\tau \frac{1}{48\, m_\tau^2} \Bigg(6 m_\tau \bar{\xi}^{E*}_{N,\tau\mu} \bar{\xi}^{E*}_{N,\tau\tau} \Big(F(z_{h^0}) - F(z_{A^0}) \Big) + m_\tau \, \bar{\xi}^{E*}_{N,\tau\mu} \bar{\xi}^{E}_{N,\tau\tau} \times \\ &\quad \times \Bigg(G(z_{h^0}) + G(z_{A^0}) + \sum_{n=1}^{\infty} \frac{m_\tau^2}{(m_\tau^{\mathrm{extr}})^2} (G(z_{n,h^0}) + G(z_{n,A^0})) \Bigg) \Bigg) \,, \\ B_2 &= Q_\tau \frac{1}{48\, m_\tau^2} \Bigg(6\, m_\tau \bar{\xi}^{D}_{N,\tau\mu} \, \bar{\xi}^{D}_{N,\tau\tau} \Big(F(z_{h^0}) - F(z_{A^0}) \Big) + m_\tau \bar{\xi}^{D}_{N,\tau\mu} \bar{\xi}^{D*}_{N,\tau\tau} \times \\ &\quad \times \Bigg(G(z_{h^0}) + G(z_{A^0}) + \sum_{n=1}^{\infty} \frac{m_\tau^2}{(m_\tau^{\mathrm{extr}})^2} (G(z_{n,h^0}) + G(z_{n,A^0})) \Bigg) \Bigg) \,, \end{aligned} \tag{2.12}$$

and $c_2 = \frac{G_F^2 \alpha_{em} m_\tau^5}{32\pi^4}$. Here the amplitudes B_1 and B_2 have right and left chirality, respectively. The functions $F(w)$ and $G(w)$ in eqs. (2.10) and (2.12)are given by

$$\begin{aligned} F(w) &= \frac{w\,(3 - 4\,w + w^2 + 2\,ln\,w)}{(-1+w)^3} \,, \\ G(w) &= \frac{w\,(2 + 3\,w - 6\,w^2 + w^3 + 6\,w\,ln\,w)}{(-1+w)^4} \,, \end{aligned} \tag{2.13}$$

where $z_H = \frac{m_\tau^2}{m_H^2}$, $z_{n,H} = \frac{m_\tau^2 + (n/R)^2}{m_H^2 + (n/R)^2}$, $(m_\tau^{\mathrm{extr}})^2 = m_\tau^2 + (n/R)^2$, Q_τ is the charge of τ lepton. The Yukawa couplings $\bar{\xi}^E_{N,ij}$ appearing in the expressions are defined as $\xi^E_{N,ij} = \sqrt{\frac{4\,G_F}{\sqrt{2}}} \, \bar{\xi}^E_{N,ij}$. In our calculations we take into account only internal τ-lepton contribution since, in our assumption, the couplings $\bar{\xi}^E_{N,ij}$, $i,j = e,\mu$, are small compared to $\bar{\xi}^E_{N,\tau\, i}$ $i = e,\mu,\tau$ due to the possible proportionality of them to the masses of leptons under consideration in the vertices and we used parametrization

$$\bar{\xi}^E_{N,\tau l} = |\bar{\xi}^E_{N,\tau l}| \, e^{i\theta_l} \,, \tag{2.14}$$

with the CP violating phase θ_l to extract the complexity of these couplings. Furthermore, we take the Yukawa couplings for the interactions lepton-KK mode of lepton-KK mode of Higgs bosons (h^0 and A^0) as the same as the ones for the interactions of zero mode fields.

3. Discussion

The LFV $l_i \to l_j \gamma$ $(i \neq j)$ interactions exist at the loop level in the model III and the Yukawa couplings $\bar{\xi}^{D}_{N,ij}, i, j = e, \mu, \tau$ are the essential parameters used in the calculations of physical quantities related to those decays. The Yukawa couplings are free parameters of the theory and they can be fixed by present and forthcoming experiments. In our calculations, we assume that the Yukawa couplings $\bar{\xi}^{E}_{N,ij}$ is symmetric with respect to the indices i and j and take $\bar{\xi}^{E}_{N,ij}$, $i, j = e, \mu$, as small compared to $\bar{\xi}^{E}_{N,\tau\, i}\, i = e, \mu, \tau$ since the strength of these couplings are related with the masses of leptons denoted by the indices of them, similar to the Cheng-Sher scenerio [17]. For the $\mu \to e\gamma$ decay the Yukawa couplings $\bar{\xi}^{E}_{N,\tau\mu}$ and $\bar{\xi}^{E}_{N,\tau e}$ play the main role. The first one is restricted by using the experimental uncertainty, 10^{-9}, in the measurement of the muon anomalous magnetic moment and the upper limit of $\bar{\xi}^{E}_{N,\tau\mu}$ is predicted as 30 GeV (see [18] and references therein). For the numerical values of the Yukawa coupling $\bar{\xi}^{E}_{N,\tau e}$, we use the prediction $10^{-3} - 10^{-2}$ GeV which respects the experimental upper limit of BR of $\mu \to e\gamma$ decay, $BR \leq 1.2 \times 10^{-11}$ and predicted value of $\bar{\xi}^{E}_{N,\tau\mu} \leq 30$ GeV (see [5] for details). For the $\tau \to \mu\gamma$ decay the Yukawa couplings $\bar{\xi}^{E}_{N,\tau\tau}$ and $\bar{\xi}^{E}_{N,\tau\mu}$ play the main role and for $\bar{\xi}^{E}_{N,\tau\tau}$ we use numerical values larger than the upper limit of $\bar{\xi}^{E}_{N,\tau\mu}$.

The addition of one extra spatial dimension brings new contribution to the BR of the decays under consideration and this contribution emerges from the KK excitations of the lepton and Higgs fields. In the case that all the fields live in higher dimension [10, 12], namely 'universal extra dimension', the KK number at each vertex is conserved and, in the present processes, the additional 'lepton-KK lepton-KK Higgs' vertices appear. In our calculations we take into account such vertices and assume that the Yukawa couplings existing are the same as the ones existing in the zero-mode case. Since the extra dimension is compactified on a an orbifold such that the zero mode leptons and Higgs fields are 4-dimensional model III particles, there exist a compactification scale $1/R$, where R is the size of the extra dimension. This parameter needs to be restricted and the lower bound for inverse of the compactification radius is estimated as ~ 300 GeV [12].

In our work we predict the one UED effect on the BR of the LFV processes $\mu \to e\gamma$ and $\tau \to \mu\gamma$ in the framework of the type III 2HDM. These decays exit at least in the one loop level in the model III and the addition of one spatial dimension results in new loop diagrams induced by the KK excitations of the lepton and Higgs fields (see figure 1). In the numerical calculations we study the BR and the ratio of KK mode contributions to the zero mode ones, within a wide range of the compactification scale $1/R$ and try to estimate the effects of possible complexity of Yukawa couplings and the mass ratio of neutral Higgs bosons, h^0 and A^0.

In figure 2, we present the compactification scale $1/R$ dependence of the BR for the LFV decay $\mu \to e\gamma$ for $m_{h^0} = 85$ GeV, $m_{A^0} = 95$ GeV, $\bar{\xi}^{D}_{N,\tau\mu} = 30$ GeV, for four different values of the coupling $\bar{\xi}^{D}_{N,\tau e}$. The solid (dashed, small dashed, dotted) line represents the BR for $\bar{\xi}^{D}_{N,\tau e} = 0.5 \times 10^{-3} (1.0 \times 10^{-3}$, 0.5×10^{-2}, $1.0 \times 10^{-2})$ GeV. This figure shows that the BR is not sensitive to the scale $1/R$ and therefore the contribution due to the one spatial extra dimension is suppressed. In the BR the main contribution comes from the

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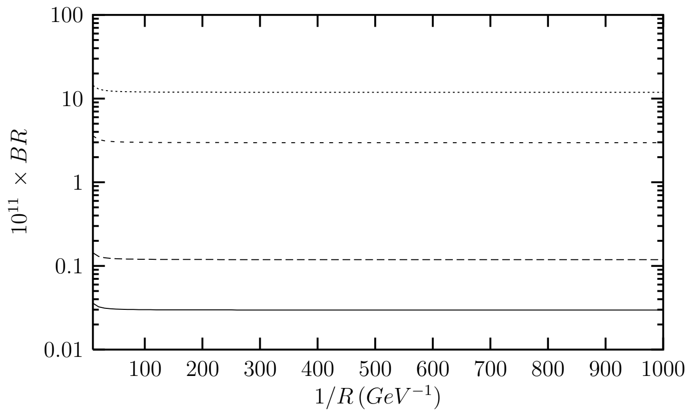

Figure 2: The compactification scale $1/R$ dependence of the BR for the LFV decay $\mu \to e\gamma$ for $m_{h^0} = 85\,\mathrm{GeV}$, $m_{A^0} = 95\,\mathrm{GeV}$, $\bar{\xi}^{D}_{N,\tau\mu} = 30\,\mathrm{GeV}$, for four different values of the coupling $\bar{\xi}^{D}_{N,\tau e}$. The solid (dashed, small dashed, dotted) line represents the BR for $\bar{\xi}^{D}_{N,\tau e} = 0.5 \times 10^{-3}(1.0 \times 10^{-3}, 0.5 \times 10^{-2}, 1.0 \times 10^{-2})\,\mathrm{GeV}$.

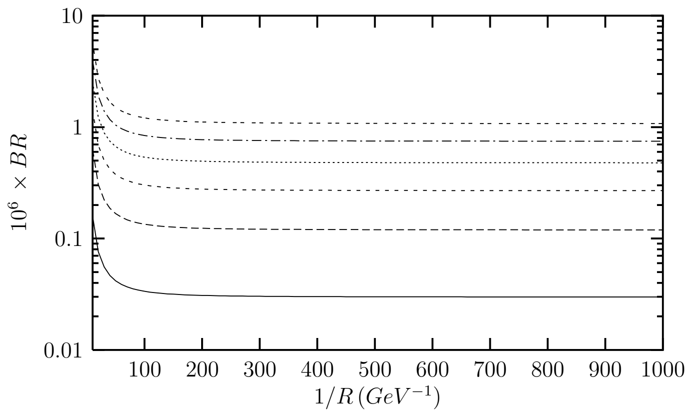

Figure 3: The compactification scale $1/R$ dependence of the BR for the LFV decay $\tau \to \mu\gamma$ for $m_{h^0} = 85\,\mathrm{GeV}$, $m_{A^0} = 95\,\mathrm{GeV}$, $\bar{\xi}^{D}_{N,\tau\mu} = 30\,\mathrm{GeV}$, for six different values of the coupling $\bar{\xi}^{D}_{N,\tau\tau}$. The solid (dashed, small dashed, dotted, dot-dashed, double dotted) line represents the BR for $\bar{\xi}^{D}_{N,\tau\tau} = 50\,(100, 150, 200, 250, 300)\,\mathrm{GeV}$.

term which is proportional to the one including the function F (see eq. (2.10)). However, the term including the extra dimension contribution is proportional the suppression factor $x_{\mu\tau} = \frac{m_\mu}{m_\tau}$. In addition to this the large compactification scale $1/R$ causes to decrease the extra dimension contribution.

Figure 3 is devoted to the compactification scale $1/R$ dependence of the BR for the LFV decay $\tau \to \mu\gamma$, for $m_{h^0} = 85\,\mathrm{GeV}$, $m_{A^0} = 95\,\mathrm{GeV}$, $\bar{\xi}^{D}_{N,\tau\mu} = 30\,\mathrm{GeV}$, for six different values of the coupling $\bar{\xi}^{D}_{N,\tau\tau}$. The solid (dashed, small dashed, dotted, dot-dashed, double dotted) line represents the BR for $\bar{\xi}^{D}_{N,\tau\tau} = 50\,(100, 150, 200, 250, 300)\,\mathrm{GeV}$. In this figure, it is shown that the BR is not sensitive to the scale $1/R$ for its large values. In the BR of this decay, the term including the extra dimension contribution does not have the suppression factor $x_{\mu\tau}$ and therefore the sensitivity of the BR is greater for small values

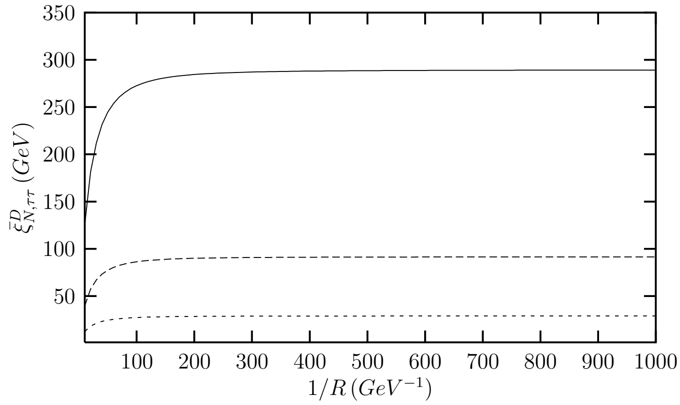

Figure 4: The relative behaviors of the coupling $\bar{\xi}^{D}_{N,\tau\tau}$ and the compactification scale $1/R$ for the fixed values of the upper limits BR of the decay $\tau \to \mu\gamma$, for $m_{h^0} = 85\,\mathrm{GeV}$, $m_{A^0} = 95\,\mathrm{GeV}$, $\bar{\xi}^{D}_{N,\tau\mu} = 30\,\mathrm{GeV}$. The solid (dashed, small dashed) line represents the upper limit of the BR as $10^{-6}\,(10^{-7},\,10^{-8})$.

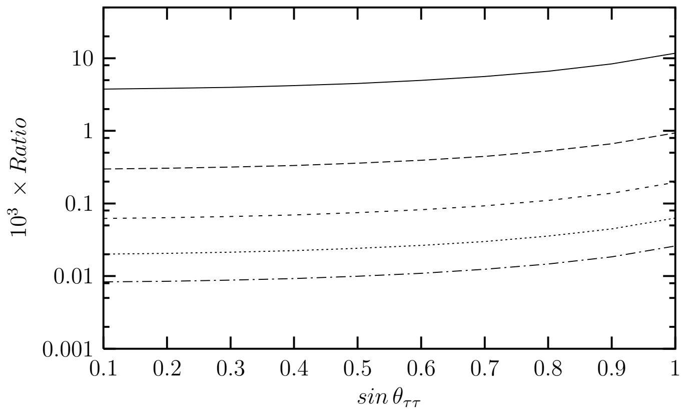

Figure 5: $sin\theta_{\tau\tau}$ dependence of the $Ratio = BR^{\mathrm{extr}}/BR^{2HDM}$, for $m_{A^0} = 95\,\mathrm{GeV}$, for five different values of the scale $1/R$. The solid (dashed, small dashed, dotted,dot-dashed) line represents the *Ratio* for $1/R = 100\,(200,\,300,\,400,\,500)\,\mathrm{GeV}$.

of the compactification scale, compared to the one for the $\mu \to e\gamma$ decay. However, at the large scale $1/R$ the extra dimension contribution is negligible.

In figure 4, we present the relative behaviors of the coupling $\bar{\xi}^{D}_{N,\tau\tau}$ and the compactification scale $1/R$ for the fixed values of the upper limits BR of the decay $\tau \to \mu\gamma$, for $m_{h^0} = 85\,\mathrm{GeV}$, $m_{A^0} = 95\,\mathrm{GeV}$, $\bar{\xi}^{D}_{N,\tau\mu} = 30\,\mathrm{GeV}$. The solid (dashed, small dashed) line represents the upper limit of the BR as $10^{-6}\,(10^{-7},\,10^{-8})$. It is observed that the increasing values of the the scale $1/R$ forces the Yukawa coupling to increase to be able to reach the numerical value of the upper limit for the BR of the process and the sensitivity to the scale $1/R$ increases with the increasing values of the upper limit of the BR.

At this stage we take the Yukawa coupling $\bar{\xi}^{D}_{N,\tau\tau}$ complex (see eq. (2.14)) and study the effects of the complexity of this coupling and the mass ratio of neutral Higgs bosons, h^0 and A^0, to the ratio of KK mode contributions to the zero mode ones, $Ratio = \frac{BR^{\mathrm{extr}}}{BR^{2HDM}}$.

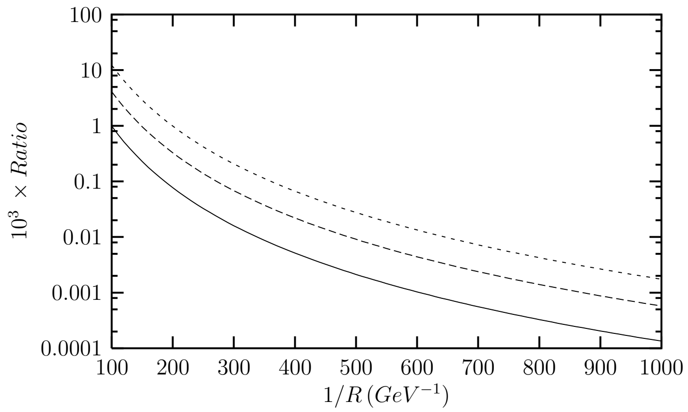

Figure 6: The compactification scale $1/R$ dependence of the Ratio for $sin\theta_{\tau\tau} = 0$ and $m_{A^0} = 95\,\mathrm{GeV}$. The solid (dashed, small dashed) line represents the dependence for the mass ratio $r = \frac{m_{h^0}}{m_{A^0}} = 0.8\,(0.9, 0.95)$.

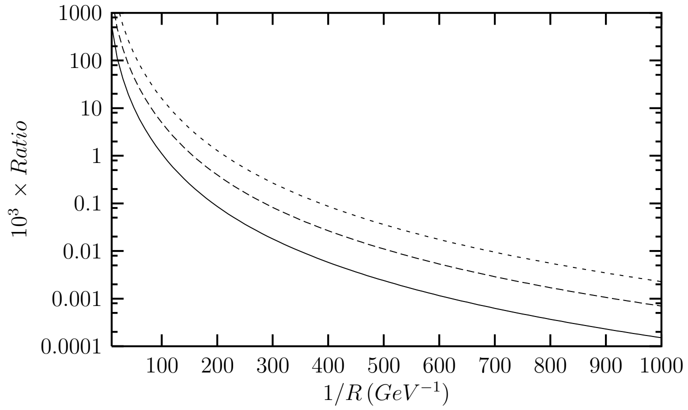

Figure 7: The same as figure 6 but for $sin\theta_{\tau\tau} = 0.5$.

Figure 5 represents the parameter $sin\theta_{\tau\tau}$ dependence of the ratio $Ratio$ where BR^{extr} (BR^{2HDM}) is the contribution of the extra dimension effects (the contribution without the extra dimensions), for $m_{A^0} = 95\,\mathrm{GeV}$ and for five different values of the scale $1/R$. The solid (dashed, small dashed, dotted,dot-dashed) line represents the $Ratio$ for $1/R = 100\,(200,\ 300,\ 400,\ 500)\,\mathrm{GeV}$. This ratio is of the order of $10^{-4} - 10^{-3}$ for the intermediate values of the scale $1/R$ and slightly increases with the increasing complexity of the Yukawa coupling $\bar{\xi}^{D}_{N,\tau\tau}$.

Finally, in figure 6 (7 and 8) we present the compactification scale $1/R$ dependence of the $Ratio$ for $sin\theta_{\tau\tau} = 0$ ($sin\theta_{\tau\tau} = 0.5$, $sin\theta_{\tau\tau} = 1$) and for $m_{A^0} = 95\,\mathrm{GeV}$. The solid (dashed, small dashed) line represents the dependence for the mass ratio $r = \frac{m_{h^0}}{m_{A^0}} = 0.8\,(0.9, 0.95)$. These figures show that the $Ratio$ increases with the increasing mass degeneracy of neutral Higgs bosons h^0 and A^0, especially for the increasing values of the complexity of the Yukawa coupling. This is an interesting result since the contribution comes from the extra dimensions are sensitive to the mass ratio of neutral Higgs bosons

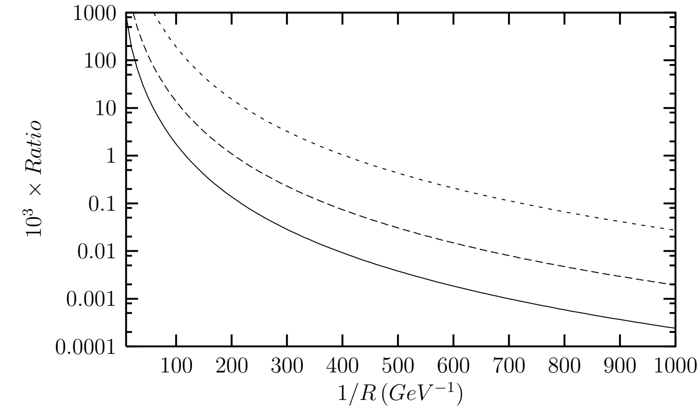

Figure 8: The same as figure 6 but for $sin\theta_{\tau\tau} = 1$.

h^0 and A^0 and also the complexity of the Yukawa coupling. Notice that the extra dimension contributions reach almost to the ordinary ones if the masses of h^0 and A^0 are nearly degenerate and the complexity of the coupling is high.

Now we would like to present the results briefly.

- The BR of $\mu \to e\gamma$ decay is not sensitive to the scale $1/R$ since the term including the extra dimension contribution is proportional the suppression factor $x_{\mu\tau} = \frac{m_\mu}{m_\tau}$ and the large compactification scale $1/R$ causes to decrease the extra dimension contribution. The BR of $\tau \to \mu\gamma$ decay is not sensitive to the scale $1/R$ for its large values. In the BR of this decay, the term including the extra dimension contribution does not have the suppression factor $x_{\mu\tau}$ and therefore the sensitivity of the BR is stronger for small values of the compactification scale, compared to the one of the $\mu \to e\gamma$ decay.

- The contribution comes from the extra dimensions is sensitive to the mass ratio of neutral Higgs bosons h^0 and A^0 and also the complexity of the Yukawa coupling.

Finally, it is not easy to investigate the extra dimension effects in the LFV decays $BR(\mu \to e\gamma)$ and $BR(\tau \to \mu\gamma)$, however, the more accurate future experimental results of these decays, hopefully, will be helpful in the determination of the signals coming from the extra dimensions.

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